Vector eq line $\Rightarrow$ Standard Form (Cartesian)

$ax + by = c$

$a, b, c \in$ integers

element

"2-D"

$$\begin{pmatrix} x \\ y \end{pmatrix} = \underbrace{\begin{pmatrix} 2 \\ 5 \end{pmatrix}}_{\text{point}} + p \underbrace{\begin{pmatrix} -3 \\ 4 \end{pmatrix}}_{\text{vector}}$$

Algebra $-\frac{4}{3} = m$ $(2,5)$

$$y - y_1 = m(x - x_1)$$

$$3(y-5) = \frac{\cancel{3}}{1} \cdot \frac{-4}{\cancel{3}}(x-2)$$

$$3y - 15 = -4x + 8$$

$$4x + 3y = 23$$

Vector Way

$$\begin{pmatrix} 4 \\ 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}\begin{pmatrix} 4 \\ 3 \end{pmatrix} + p\begin{pmatrix} -3 \\ 4 \end{pmatrix}\begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

$$4x + 3y = 8 + 15$$

$$4x + 3y = 23$$

Vector way parametric form

$x = 2 - 3p$ $\qquad$ $y = 5 + 4p$

$\frac{x-2}{-3} = p$ $\qquad$ $\frac{y-5}{4} = p$ $\qquad$ $\frac{x-2}{-3} = \frac{y-5}{4}$

$4x - 8 = -3y + 15$

$4x + 3y = 23$

Velocity Vector

Ex) A drone is moving w/a speed $15 \frac{km}{hr}$ or $15 km\,hr^{-1}$ in the direction $\langle 0, -3, 4 \rangle$ Write a velocity vector

To make velocity vector:

a) $|a| = \sqrt{0^2 + (-3)^2 + (4)^2}$
$= \sqrt{9+16} = \sqrt{25} = 5$

b) make unit vector
$\left\langle \frac{0}{5}, \frac{-3}{5}, \frac{4}{5} \right\rangle$

c) multiply by speed
$15 \left\langle 0, \frac{-3}{5}, \frac{4}{5} \right\rangle$
$\langle 0, -9, 12 \rangle$

IB Vector Worksheet Day 3 CW (M07/SP2/Q#4) Name:_____ Group # _____

Points P & Q have position vectors $-5\vec{i}+11\vec{j}-8\vec{k}$ and $-4\vec{i}+9\vec{j}-5\vec{k}$ respectively and both lie on line L_1

1. Find $\overrightarrow{PQ}$

Pt P $(-5, 11, -8)$
Pt Q $(-4, 9, -5)$

End − Start

$\overrightarrow{PQ} = \begin{pmatrix} -4 - -5 \\ 9 - 11 \\ -5 - -8 \end{pmatrix} \rightarrow \overrightarrow{PQ} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$

2. Hence, show that the equation of L_1 can be written as $\vec{r} = (-5+t)\vec{i} + (11-2t)\vec{j} + (-8+3t)\vec{k}$

$\vec{r} = \begin{pmatrix} -5 \\ 11 \\ -8 \end{pmatrix} + t\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$

direction vector

POINT P
$\rightarrow \overrightarrow{OP}$

3. The point $R(2, y_1, z_1)$ also lies on L_1. Find the value of y_1 *and* z_1.

x y z

$\boxed{R(2,-3,13)}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow \begin{pmatrix} 2 \\ y_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} -5 \\ 11 \\ -8 \end{pmatrix} + t\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \rightarrow$

$2 = -5 + t \rightarrow \boxed{t = 7}$

$y_1 = 11 - 2(7) \rightarrow y_1 = -3$

$z_1 = -8 + 3(7) \rightarrow z_1 = 13$

The line L_2 has equation $\vec{r} = 2\vec{i} + 9\vec{j} + 13\vec{k} + \overset{M}{\cancel{t}}(\vec{i} + 2\vec{j} + 3\vec{k})$

4. The lines L_1 and L_2 intersect ~~at a point~~ T. Find the position vector of T.

$L_1: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -5 \\ 11 \\ -8 \end{pmatrix} + t\boxed{\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}} \rightarrow$ $x = -5 + t$, $y = 11 - 2t$, $z = -8 + 3t$

$L_2: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 9 \\ 13 \end{pmatrix} + M\boxed{\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}} \rightarrow$ $x = 2 + M$, $y = 9 + 2M$, $z = 13 + 3M$

$x: -5 + t = 2 + M \rightarrow t - M = 7$

$y: 11 - 2t = 9 + 2M \rightarrow -2t - 2M = -2$

$z: -8 + 3t = 13 + 3M \rightarrow 3t - 3M = 21$

$\begin{cases} t - M = 7 \\ -2t - 2M = -2 \\ 3t - 3M = 21 \end{cases} \xrightarrow{(-\frac{1}{2})} \begin{aligned} t - M &= 7 \\ t + M &= 1 \\ \hline 2t &= 8 \\ t &= 4 \\ M &= -3 \end{aligned}$

MUST check if skew

$3(4) - 3(-3) = 21$ ✓
$12 + 9 = 21$
∴ NOT skew!

$t = 4$

$x = -5 + (4) \rightarrow -1$
$y = 11 - 2(4) \rightarrow 3$
$z = -8 + 3(4) \rightarrow 4$

POINT $(-1, 3, 4)$

$\boxed{\begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix}}$

5. Calculate the angle between the lines L_1 and L_2

$\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$

$\cos\theta = \dfrac{\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}}{\left|\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}\right| \left|\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}\right|}$

$\cos\theta = \dfrac{1(1) + (-2)(2) + 3(3)}{\sqrt{1^2 + -2^2 + 3^2}\sqrt{1^2 + 2^2 + 3^2}}$

$\cos\theta = \dfrac{6}{\sqrt{14}\sqrt{14}}$

$\cos\theta = \dfrac{6}{14}$

$\theta = \cos^{-1}\left(\dfrac{6}{14}\right)$

$\boxed{\theta \approx 64.62^\circ}$