Int3MA 10.2.2 Infinite Series & friends Name: Key Group #____ Period:____

1. On back, use mathematical induction to prove the following statement. $-1+2+5+8+\ldots+3n-4=\frac{n(3n-5)}{2}$

2. A ball is tossed up 10 meters. Instead of catching the ball, it bounces 60% of its previous height.

a. Write a geometric series.

UP $10+6+3.6+\cdots$ OR $20+12+7.2+\cdots$

DOWN $10+6+3.6+\cdots$

b. Find the total distance the ball travels.

$2\cdot S \to 2\left(\frac{10}{1-.6}\right) \to 2(25)$

$\boxed{50}$

3. Write the sum using sigma notation and then find its value.

a. $\frac{1}{3}+\frac{2}{3}+1+\frac{4}{3}+\ldots+33$

$a_n=\frac{1}{3}n$

$33=\frac{1}{3}n \to n=99$

$\sum_{n=1}^{99}\left(\frac{1}{3}n\right) \to \frac{99}{2}\left[\frac{1}{3}+33\right] \to \boxed{1650}$

b. $\frac{1}{625}+\frac{1}{125}+\frac{1}{25}+\ldots$

$\sum_{n=1}^{\infty}\left(\frac{1}{625}\right)(5)^{n-1}$

$\boxed{\text{No Sum } r>1}$

4. Find the given sum, or explain why it does not exist.

a. $1-\frac{2}{3}+\frac{4}{9}-\frac{8}{27}+\ldots$

$r=-\frac{2}{3}$

$S=\frac{1}{1-(-\frac{2}{3})}$

$S=\boxed{0.6}$

b. $\frac{1}{3}+\frac{\sqrt{2}}{3}+\frac{2}{3}+\frac{2\sqrt{2}}{3}+\ldots$

$r=\sqrt{2}$ $r>1$

Sum $\to \infty$

Does not Exist

5. Find the sum algebraically.

a. $\sum_{j=6}^{8}(10-2j)$

$S_3=\frac{3}{2}\left[-2+{}^{-}6\right]$

$=\boxed{-12}$

b. $\sum_{k=0}^{3}\frac{3}{2}(2)^k$

$S_4=\frac{\frac{3}{2}\left(1-(2)^4\right)}{(1-2)}$

$S_4=\boxed{22.5}$

6. Find n <u>algebraically</u>.

a. 325 + 130 + 52 + ... $a_n \approx 1.3312$

$r=0.4$

$a_n=325(0.4)^{n-1}$

$1.3312=325(0.4)^{n-1}$

$0.004096=0.4^{n-1}$ Change to LOG form OR Take LOG of both sides

$\log_{0.4}(0.004096)=n-1$

$6=n-1$

$\boxed{n=7}$

b. $S_n=-294$ for $\sum_{n=1}^{\infty}(9-4n)$ Arithmetic

$S_n=\frac{n}{2}(\underbrace{5}_{\text{First term}}+\underbrace{9-4n}_{n\text{th term}})$

$-294=\frac{n}{2}(-4n+14)$

$-588=n(-4n+14)$

$4n^2-14n-588=0$

$n=\frac{14\pm\sqrt{196-4(4)(-588)}}{8}$

$n=\frac{14\pm 98}{8}$ $\to \boxed{14}$ $\to -10.5$

7. Find a_n algebraically for an <u>arithmetic</u> series in which $a_7=20, a_{15}=44$.

Either systems OR

$a_7=20 \to 20=a_1+6d$

$a_{15}=44 \to 44=a_1+14d$

$-24=-8d$

$d=3$

$20=a_1+6(3)$

$a_1=2$

$\boxed{a_n=2+(n-1)(3)}$

last first

$a_n=a_1+(n-1)d$

$44=20+(15-7)d$

$24=8d$

$d=3$

$20=a_1+(6)(3)$

8. Find a_n for a <u>geometric</u> series in which $a_5=4$ and $a_7=12$.

$a_5=4 \to 4=a_1\cdot r^4 \to a_1=\frac{4}{r^4}$

$a_7=12 \to 12=a_1\cdot r^6 \to 12=\frac{4}{r^4}\cdot r^6 \to$ $12=4r^2$, $3=r^2$, $r=\sqrt{3}$

$4=a_1\cdot r^4$

$4=a_1(\pm\sqrt{3})^4$

$\frac{4}{9}=a_1$

$\boxed{a_n=\left(\frac{4}{9}\right)(\sqrt{3})^{n-1}}$

Prove true $\forall n \geq 1$

$$-1+2+5+8+\cdots+(3n-4)=\frac{n(3n-5)}{2}$$

Prove true $n=1$

$$-1=\frac{1(3(1)-5)}{2} \longrightarrow -1=\frac{-2}{2} \rightarrow -1=-1$$

☺ ✓

Assume true $n=k$

$$-1+2+5+8+\cdots+(3k-4)=\frac{k(3k-5)}{2}$$

Prove true $n=k+1$

$$\underbrace{-1+2+5+8+\cdots+(3k-4)}+(3(k+1)-4)=\frac{(k+1)(3(k+1)-5)}{2}$$

$$\frac{k(3k-5)}{2}+\frac{2(3k+3-4)}{2}=$$

$$\frac{3k^2-5k+6k+6-8}{2}=$$

$$\frac{3k^2+k-2}{2}=$$

$$\frac{(3k-2)(k+1)}{2}=$$

$$\frac{(k+1)(3(k+1)-5)}{2}=\frac{(k+1)(3(k+1)-5)}{2}$$

QED