3.1.4 How can I move it?

Defining Rigid Transformations

Photo courtesy of the artist.

Sue Sales, *Hearts*.

Throughout American history, quilters have used transformations to create intricate geometric designs. For example, the quilt at right is an example of a design based on rotation and reflection, while the quilt at left contains translations, rotations, and reflections.

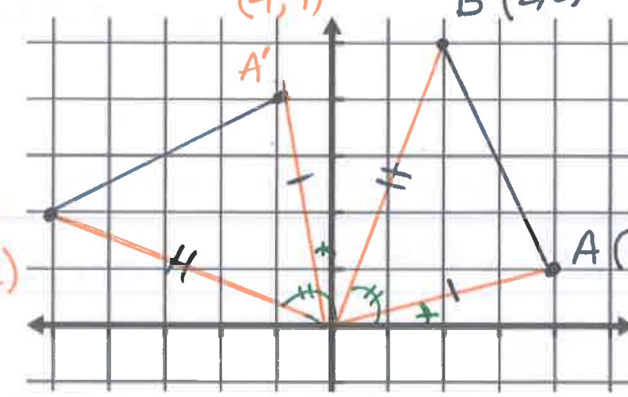
Photo courtesy of the artist.

Sue Sales, *Balance From Within*.

In Lesson 3.1.3, you found ways to locate the image of a shape after it is reflected. Today, you will work with your team to develop ways to describe the image of a shape after it is rotated or translated.

3-39. DEFINITION OF ROTATION

So what exactly is a rotation? If a figure is rotated, how can you describe it? Investigate this question below.



a. On the graph, graph the points $A(4, 1)$ and $B(2, 5)$. Then use tracing paper to rotate the two points 90° counterclockwise (↺) about point $O(0, 0)$. Mark points A' and B' on your graph paper.

b. Draw line segments that connect A to O, B to O, A' to O, and B' to O. Using tracing paper, compare the lengths of, and angles formed by, these line segments. Which lengths are equal? Which angle measures are equal? Be specific.

$\overline{OA} \cong \overline{OA'}$

$\overline{OB} \cong \overline{OB'}$

LENGTHS ARE EQUAL between $\overline{OA}$ $\overline{OA'}$ between $\overline{OB}$ $\overline{OB'}$

$m\angle AOB = m\angle A'OB'$

c. Predict $m\angle AOA'$. Verify its measure.

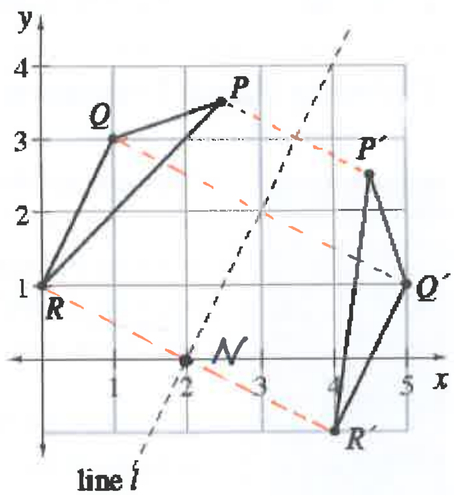
$= 90°$

d. Definition of Rotation:

A rotation preserves the distance between all points and the point of rotation.

3-40. DEFINITION OF REFLECTION

Now that you know more about the slopes of parallel and perpendicular lines, revisit the reflection from problem 3-25 and confirm the relationships you found in that problem as follows.



a. The diagram at right shows ΔPQR reflected across line l to form $\Delta P'Q'R'$. Use your ruler to draw three dashed line segments: $\overline{PP'}$, $\overline{QQ'}$, and $\overline{RR'}$. What is the relationship between these three dashed line segments? Use your knowledge of slope to algebraically verify your observations.

– All lines --- are parallel $m = -\frac{1}{2}$

b. Now focus only on line segment $\overline{RR'}$. What is the relationship between the line of reflection and segment $\overline{RR'}$?

They look to be PERPENDICULAR

c. Use slope to confirm that the line of reflection is perpendicular to line segment $\overline{RR'}$.

$\overline{RR'} \rightarrow m = -\frac{1}{2}$ line $l \rightarrow m = \frac{2}{1}$ Yes, they are perpendicular b/c their slopes are opposite reciprocals.

d. Place a point N at the intersection of $\overline{RR'}$ and the line of reflection. What do you notice about the lengths of segments $\overline{RN}$ and $\overline{NR'}$?

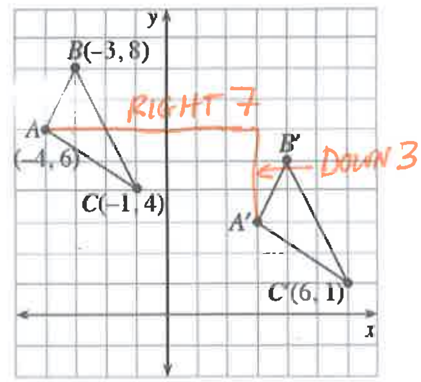
$m\overline{RN} = m\overline{NR'}$ EQUAL lengths!

e. Define a reflection:

A reflection must have equal lengths from each point x' to its preimage x.

3-42. DEFINITION OF TRANSLATION

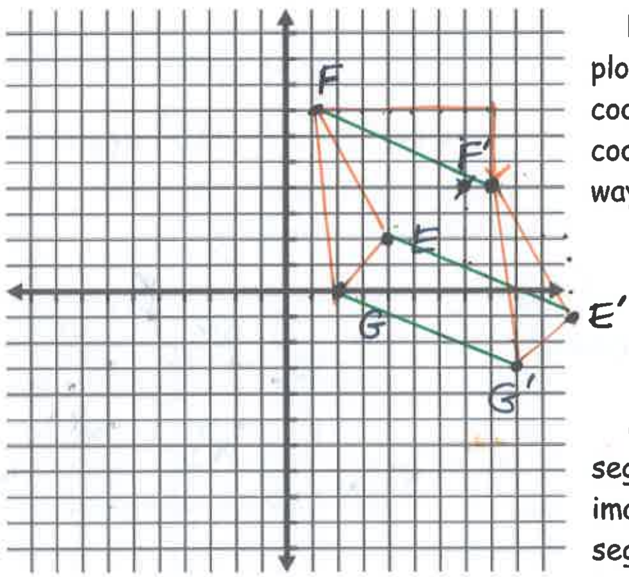
The formal name for a slide is a translation. So what exactly is a translation? (Remember that translation and transformation are different words.) $\Delta A'B'C'$ at right is the image of a translation of ΔABC.



a. Describe the translation. That is, how many units to the right and how many units down did the translation move the triangle?

RIGHT 7, DOWN 3

$(x, y) \rightarrow (x+7, y-3)$



b. On the graph at left, plot ΔEFG with coordinates $E(4, 2)$, $F(1, 7)$, and $G(2, 0)$. What are the coordinates of $\Delta E'F'G'$ if $\Delta E'F'G'$ is translated the same way as ΔABC was in part (a)?

$F'(8, 4)$

$E'(11, -1)$

$G'(9, -3)$

c. For the translated triangle in part (b), draw line segments connecting each vertex to its translated image. What do you notice about these line segments? What does this tell you about how a translation moves each point of the triangle?

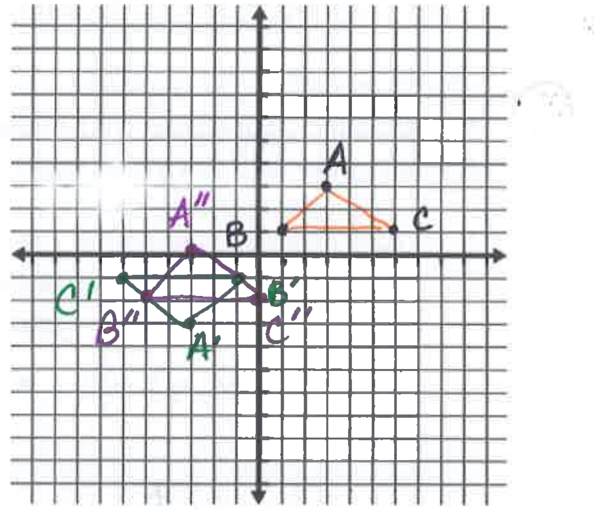
Each of these line segments (in green) are parallel and equal in length!!

$\boxed{m = \frac{-3}{7}}$

d. Define translation

The shape is NOT changed but shifted R/L ~~or~~ and U/D so that all points are moved a FIXED distance from each other and angles are not altered.

3-43. CONNECTIONS TO ALGEBRA



Read the Math Notes box in this lesson. Note that the formal definitions of transformations involve functions.

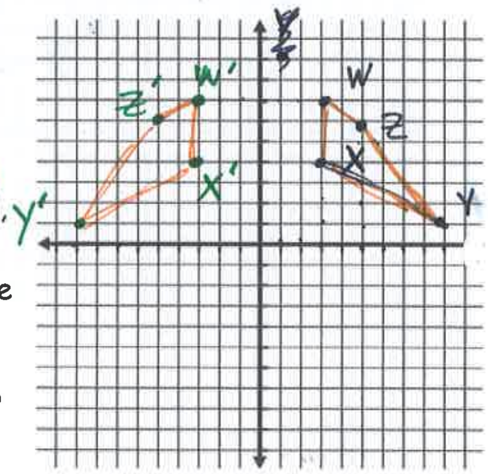
a. Plot ΔABC on graph paper with points $A(3, 3)$, $B(1, 1)$, and $C(6, 1)$. Apply the function $(x, y) \rightarrow (-x, -y)$ to find the coordinates of the image $\Delta A'B'C'$. For example, using the given function, the original point $(8, -3)$ has an image point $(-8, 3)$.

A' (-3,-3)
B' (-1,-1)
C' (-6,-1)

b. What single transformation is the function in part (a)? A rotation of 180° about the origin

c. Use the function $(x, y) \rightarrow (x - 6, y - 3)$ to transform the triangle ΔABC again. Name the coordinates of the image $\Delta A''B''C''$, and describe the transformation (or sequence of transformations).

A'' (-3,0)   Left 6
B'' (-5,-2)   Down 3
C'' (0,-2)



d. On a new set of axes, plot and connect the points to form quadrilateral $WXYZ$ if its vertices are $W(3, 7)$, $X(3, 4)$, $Y(9, 1)$, and $Z(5, 6)$. Quadrilateral $WXYZ$ is transformed by the function $(x \rightarrow -x, y \rightarrow y)$ to form quadrilateral $W'X'Y'Z'$. Where is Y'?

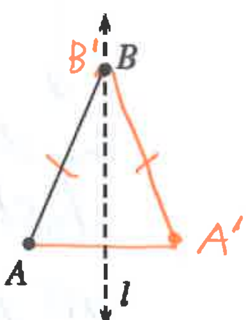
Y'(-9,1)

e. What transformation mapped quadrilateral $WXYZ$ to its image?

A reflection across the Y-AXIS

3-44. FACTS ABOUT ISOSCELES TRIANGLES

How can transformations such as reflections help us to learn more about familiar polygons? Consider reflecting a line segment across a line that passes through one of its endpoints.



a. On the diagram at right, draw $\overline{A'B}$, the reflection of $\overline{AB}$ across line l. When points A and A' are connected, what special polygon is formed by points A, B, and A'?

An isosceles triangle ... Δ with 2 equal sides

b. Use what you know about reflection to make as many statements as you can about the triangle formed in part (a). For example, are there any sides that must be the same length? Are there any angles that must be equal? Is there anything else special about this polygon?

Δ has 2 EQUAL sides due to reflection
Angles opposite the EQUAL sides are equal
the line of reflection divides the third side into 2 equal pies
AND its perpendicular to that side.